

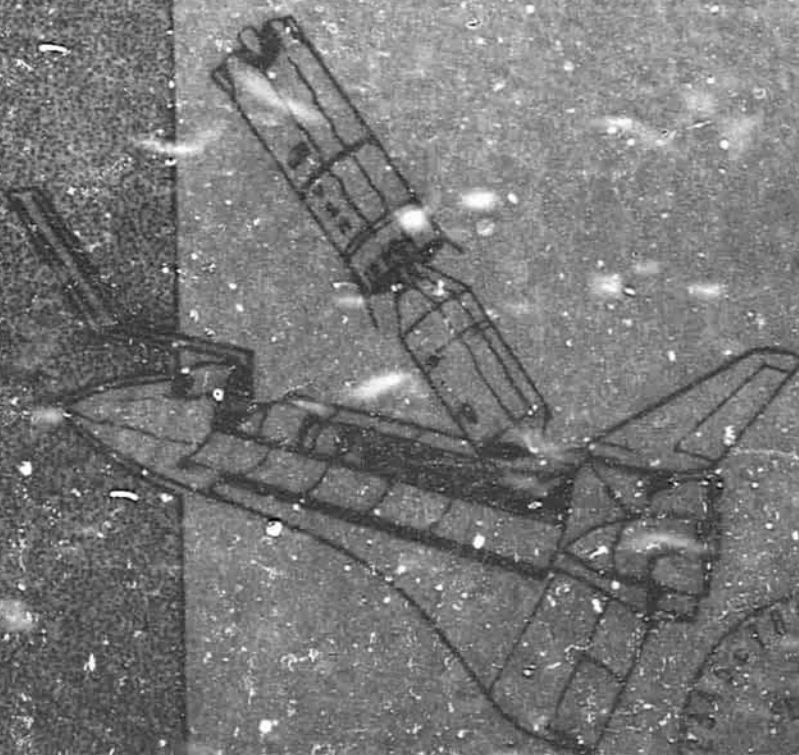
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ASYMMETRICAL BOOSTER ASCENT
GUIDANCE AND CONTROL
SYSTEM DESIGN STUDY

VOLUME V

SPACE SHUTTLE POWERED
EXPLICIT GUIDANCE



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SYSTEM DESIGN STUDY

VOLUME V

SPACE SHUTTLE POWERED EXPLICIT GUIDANCE

JUNE 28, 1974

PREPARED BY

R.F. Jagers
R. F. JAGGERS

APPROVED BY

W. G. Ryals
W. G. RYALS
PROGRAM MANAGER

PREFACE

Final report of Asymmetrical Booster Ascent Guidance and Control System Design Studies performed under Contract NAS9-13568 are contained in five separate volumes identified as follows:

- Volume I - Summary
- Volume II - SSFS Math Models - Ascent
- Volume III - Space Shuttle Vehicle SRB Actuator Failure Study
- Volume IV - Sampled Data Stability Analysis Program (SADSAP) -
Users Guide
- Volume V - Space Shuttle Powered Explicit Guidance

ABSTRACT

This report was prepared under contract NAS9-13568 for the Guidance and Control Systems Branch of the Avionics Systems Engineering Division, Lyndon B. Johnson Space Center. The report presents an optimum powered explicit guidance algorithm capable of handling all Shuttle exoatmospheric maneuvers. Theoretical and practical basis for the currently baselined Shuttle powered flight guidance equations and logic, as documented by MIT, is presented. The development is based on previous work by JSC, MIT, MSFC, Boeing and others. Detailed flow diagrams for implementing the steering computations for all Shuttle phases, including powered Return to Launch Site (RTL) abort, are presented. Derivation of the powered RTL algorithm is presented, as well as detailed flow diagrams for implementing the option. The flow diagrams and equations are compatible with the current MIT Shuttle powered flight document.

KEY WORDS

Guidance Algorithm
Saturn Iterative Guidance Mode (IGM)
Apollo Guidance
Explicit Guidance
Shuttle Powered Explicit Guidance
Orbiter Main Engine Cutoff (MECO)
Orbiter Maneuvering System (OMS)
Return to Launch Site (RTL) Abort Guidance
Propellant Depletion
Range Throttling

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The author gratefully acknowledges the work of Helmut J. Horn, Daniel T. Martin, the late Issac E. (Gene) Smith, and Judson J. Hart, former colleagues of the author at the NASA Marshall Space Flight Center, who used much physical insight in conceiving and developing the Saturn Iterative Guidance Mode (IGM), from which the Linear Tangent Guidance (LTG) concept evolved. Horn conceived the IGM approach as a "quick-look" performance tool, Martin and Smith used much ingenuity in developing and implementing the concept, and Hart recognized the guidance potential of the tool. The author, taking advantage of the physical insight of Horn, Martin, Smith and Hart, developed the LTG approach mathematically. The author expresses appreciation to A. David Long and R. Leroy McHenry of the Johnson Space Center who recognized the potential of the original ascent LTG to be extended to handle the various Shuttle powered flight phases (including abort), and who have made many other significant contributions to the implementation of the unified LTG approach. The author expresses appreciation to Timothy J. Brand, Dennis W. Brown, and John P. Higgins of the Massachusetts Institute of Technology who developed concepts and techniques that enable the Shuttle ascent Linear Tangent Guidance to be extended to execute all vacuum-flight thrusting maneuvers. The author also expresses appreciation to Donald J. Jezewski of the Johnson Space Center who developed an ingenious closed-form method of eliminating all flat earth assumptions in the theoretical development of the guidance problem.

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1.0 INTRODUCTION

The original powered flight guidance software proposed for Shuttle consisted of various specialized routines to handle the various phases of flight (as Saturn V/Apollo). The Boeing developed Linear Tangent Guidance (LTG) equations (Reference 1) were baselined for Shuttle nominal ascent. Investigation revealed that Apollo type guidance was inadequate to handle some phases of Shuttle flight (i.e., low thrust deorbit). It was later discovered by Long and McHenry of the Mission Planning and Analysis Division (MPAD) of JSC that the basic LTG algorithm was accurate and flexible enough to handle some of these difficult phases. Long and McHenry (Reference 3) then developed the concept of extending the basic ascent LTG equations to handle all phases of Shuttle powered flight, including Abort Once Around (AOA) and Return To Launch Site (RTLS) abort. Brand, Brown, and Higgins (Reference 2) of the Massachusetts Institute of Technology (MIT) later developed concepts that made it feasible to extend the ascent LTG to handle various phases of Shuttle flight.

The basic Boeing guidance task under this contract was to coordinate with MPAD, MIT, and Rockwell International (RI) in development of the Unified Linear Tangent Guidance (ULTG) and to implement this capability in the JSC Space Shuttle Functional Simulator (SSFS). Emphasis was placed on abort capability. Various candidate abort techniques were critiqued. It was determined that all of the existing candidates for RTLS abort guidance had certain limitations and disadvantages. As a result, Boeing developed a complete solution to the RTLS abort problem satisfying all constraints and compatible with the ULTG as a simple option (Section 5.2). In order to solve the RTLS problem, the original LTG algorithm was significantly

modified. A unified optimum guidance algorithm (References 8 and 13), involving higher order accuracy and more flexibility, was developed. This accuracy and flexibility made range throttling possible (Section 4.2) and made it possible for the steering to modulate to deplete excess propellant and simultaneously satisfy velocity and position constraints (Section 5.2).

The guidance work performed under this contract contributes to all phases of Shuttle powered flight guidance. A technique for handling the Orbiter Maneuvering System (OMS) ascent phase (Reference 9) was developed and is presented in Section 4.1. This technique has been implemented in the ULTG of the SSFS and incorporated into the currently baselined Shuttle powered flight guidance equations (Reference 10). Theoretical background and development of the unified optimum guidance algorithm are presented in Sections 2.0 and 3.0. Techniques for optimally handling various sets of end-conditions are presented in Section 4.0. (Techniques that were undeveloped for Saturn/Apollo technology). Two explicit powered flight guidance algorithms for RTLS are presented in Section 5.0. Section 6.0 contains a method for implementing the steering computations for all Shuttle phases. RTLS time-to-go and range throttling equations are also included. References are listed in Section 7.0. Derivation of generalized higher order thrust integrals is presented in Appendix A. A simple gravity model capable of accurately handling all Shuttle powered maneuvers is presented in Appendix B. However, this is not the model in the currently baselined equations (Reference 10). The RTLS powered flight option is presently being programmed in the SSFS by NASA/JSC and LEC.

The guidance algorithm has the following features:

Optimality - All necessary conditions for optimality are satisfied to within the accuracy of the guidance algorithm. All flat earth assumptions are eliminated.

Accuracy - This guidance algorithm is an approximate closed-form solution to the powered flight trajectory optimization problem.

Stability - There is considerable feedback in the algorithm; i.e., everything is a function of everything else. However, in the final analysis, everything is a function of final state, costate, and final time. These final quantities converge to virtually constant values after a few cycles through guidance.

Flexibility - The algorithm can handle any reasonable set of maneuver end-conditions in an optimum manner. The number of end-conditions can vary from one to seven.

Simplicity - The basic equations are very clean and simple, e.g., cleaner and simpler than the Saturn Iterative Guidance Mode (IGM), the original LTG approach of Reference 1, and the equations of References 2 and 3.

It will be seen in the following sections that the concept presented in this report actually represents a problem solution approach. The approach, although numerical integration could be employed, eliminates expensive (solution time and computer storage) Newton-Raphson type iterations. Desired maneuver end-conditions are expressed as functions of the terminal velocity and position vectors. The navigator furnishes instantaneous

values of state, time, thrust acceleration and gravity acceleration. Analytic expressions are employed to represent the magnitude of the thrust acceleration time history. The problem is to determine the thrust vector time history (and issue steering commands and throttle command if applicable) that maximizes final mass subject to satisfying maneuver objectives (end-conditions). Theoretical development furnishes the optimum form of the thrust vector (steering parameters) and missing end-conditions necessary for optimality. The problem solution approach is to develop a closed form approximation for determining values of the steering parameters that satisfy all necessary and desired conditions. The solution is a predictor/corrector or recursive type iteration. Several pre-thrust or pre-guidance cycles through the equations are assumed necessary to converge the algorithm.

The solution approach is as follows: The solution is linearized in terms of steering parameters, thrust and gravity integrals and final state. The steering parameters are then determined from the linear equations. The steering parameters and thrust integrals are in turn used to predict final state in a manner that approximately satisfies down-range conditions. Down-range conditions already being satisfied, cross-range and radial conditions as well as appropriate magnitudes are corrected if errors exist, i.e., a discontinuity is introduced into the predicted state to satisfy end-conditions. A one pass iteration is repeated on the next periodic guidance cycle. The algorithm is extremely convergent, even for bad guess starting values of final state, costate and time. An initial estimate of velocity-to-go is needed to start the algorithm.

2.0 THEORETICAL BACKGROUND

The guidance and trajectory optimization problem fits in the class of the problem of Bolza or more specifically the problem of Mayer (Reference 5, Chapter 4). This problem can be formulated as follows: consider the class of functions

$S_i(t)$, $i = 1, \dots, n$, where t is the independent variable,

Satisfying the constraints (differential equations)

$$\phi_j(t, S_i, \dot{S}_i) = 0, j = 1, \dots, \ell, \text{ where } \dot{S} \equiv \frac{dS}{dt},$$

which involve $n-\ell$ degrees of freedom. These functions also satisfy the end conditions

$$C_k(t_1, S_{i1}) = 0, k = 1, \dots, m$$

$$C_k(t_2, S_{i2}) = 0, k = m+1, \dots, s \leq 2n+2.$$

The problem is to find that special set of these functions (S_i) that minimizes or maximizes

$$G(t, S_i) \Big|_1^2.$$

This problem can be treated in a simple and elegant manner by introducing a set of variable Lagrange multipliers,

$$\lambda_j(t), j = 1, \dots, \ell \text{ (constraints)}$$

and a set of constant Lagrange multipliers,

$$\nu_k, k = 1, \dots, s \text{ (end conditions),}$$

and employing the principles and theorems in Reference 6, Chapter 7, and Reference 7, Chapter 2.

For simplicity the notation of Reference 6 is employed here, e.g.,

$$F_{\dot{S}_i} dS_i \equiv \sum_{i=1}^n \frac{\partial F}{\partial \dot{S}_i} dS_i, \lambda_j \phi_j \equiv \sum_{j=1}^{\ell} \lambda_j \phi_j.$$

The above problem can be reduced to minimizing or maximizing the augmented functional

$$P = [G]_1^2 + \mu_k C_k + \int_{t_1}^{t_2} F dt, \text{ where } F = \lambda_j \phi_j = 0$$

In order to extremize P subject to constraints the first variation, δP , must vanish. The following commutative laws from Reference 7 are used in formulating the first variation, δP .

$$\delta \int_{t_1}^{t_2} F dx = \int_{t_1}^{t_2} \delta F dx, \quad \delta \frac{dS}{dx} = \frac{d}{dx} \delta S$$

where x is the independent variable. It is also shown in Reference 7 that

$$\delta(F_1/F_2) = (F_2 \delta F_1 - F_1 \delta F_2)/F_2^2 \text{ as in differentiation.}$$

In order to obtain all of the necessary conditions for optimality shown in References 5 and 6, the equations are written in parametric form as in Reference 6, i.e.,

$$t = x, S_i = S_i(x) \text{ from which it follows that } F = F \left[t(x), S_i(x), \frac{dS_i/dx}{dt/dx} \right] \frac{dt}{dx},$$

$$\delta F = \frac{\partial F}{\partial S_i} \delta S_i + \frac{\partial F}{\partial \dot{S}_i} \delta \dot{S}_i + \frac{\partial F}{\partial t} \delta t + \frac{\partial F}{\partial \dot{t}} \delta \dot{t} \text{ (where } \dot{t} = \frac{dt}{dx}, \frac{dS}{dt} = \frac{dS/dx}{dt/dx} \text{ and}$$

$$\delta \left(\frac{dS}{dt} \right) = [(dt/dx) \delta(dS/dx) - (dS/dx) \delta(dt/dx)] / (dt/dx)^2$$

or

$$\delta \dot{S} = \frac{d}{dt} \delta S - \frac{dS}{dt} \delta \dot{t} \text{ since } \delta \text{ and } \frac{d}{dx} \text{ are commutative, and } \delta \dot{t} = \frac{d}{dt} \delta t.$$

With the above definitions we are now ready to formulate the first variation, δP . With the usual integration by parts, removing some terms from the integral, the first variation is as follows:

$$\delta P = T + I, \text{ where}$$

$$T = [dG + (F - \dot{S}_i F_{\dot{S}_i}) dt + F_{\dot{S}_i} dS_i]_1^2 + \mu_k dC_k$$

$$I = - \int_{t_1}^{t_2} \left\{ \left(\frac{d}{dt} F_{\dot{S}_i} - F_{S_i} \right) \delta S_i + \left[\frac{d}{dt} (F - \dot{S}_i F_{\dot{S}_i}) - F_t \right] \delta t \right\} dt$$

It is necessary for optimality that the coefficients of the differentials vanish. The vanishing of the coefficients of the differentials in the expression T is called the "transversality condition". The coefficients of δS_i in the integral are called the Euler-Lagrange equations.

$$\frac{d}{dt} F_{\dot{S}_i} - F_{S_i} = 0$$

Inspection shows that

$-(F - \dot{S}_i F_{\dot{S}_i}) \equiv H$, called the Hamiltonian of the system. The coefficient of δt in the integral becomes $\dot{H} + F_t = 0$ (called the "first integral"). Consequently, for problems where F is formally independent of t, the Hamiltonian is a constant of the system.

$H = C$ (constant), and the transversality condition reduces to

$$[dG - Cdt + F_{\dot{S}_i} dS_i]_1^2 + \mu_k dC_k = 0.$$

So, in conclusion, the "transversality condition", the Euler-Lagrange equations, the "first integral", and the constraints and end conditions being satisfied form the necessary conditions for optimality in the problem of Mayer, stated above. The above necessary conditions are general and consistent with the problem statement.

3.0 NECESSARY CONDITIONS APPLIED TO GUIDANCE

For reference, the necessary conditions for optimality are summarized here in common notation.

$$\frac{d}{dt} \frac{\partial F}{\partial \dot{S}_i} - \frac{\partial F}{\partial S_i} = 0, \quad i = 1, \dots, n \quad (1)$$

$$\left[dG - H dt + \sum_{i=1}^n \frac{\partial F}{\partial \dot{S}_i} dS_i \right]_1^2 + \sum_{j=1}^s \mu_j dC_j = 0 \quad (2)$$

$$\dot{H} + \frac{\partial F}{\partial t} = 0, \text{ where} \quad (3)$$

$$F = \sum_{j=1}^l \lambda_j \phi_j \text{ and}$$

$$H = - \left(F - \sum_{i=1}^n \dot{S}_i \frac{\partial F}{\partial \dot{S}_i} \right)$$

The exoatmospheric equations of motion are

$$\dot{\underline{V}} = a_T \underline{p}(t) + \underline{G}(R) \quad (4)$$

$$\dot{\underline{R}} = \underline{V}$$

$$\dot{m} = f_7$$

Where \underline{V} is velocity, \underline{R} is radius, m is vehicle mass, and \underline{G} is gravity.

$$a_T = \frac{T}{m}, \text{ where } T \text{ is engine thrust.}$$

$$\underline{p} = \begin{bmatrix} \sin \theta \cos \psi \\ \sin \theta \sin \psi \\ \cos \theta \end{bmatrix}, \text{ a unit vector} \quad (5)$$

defining the thrust direction (when implementing the guidance law the small thrust vector control oscillations may be ignored).

An ingenious method for handling gravity in the problem formulation has been developed by Jezewski (Reference 4). The approximation is made that the gravity vector is a linear function of the radius vector, i.e.

$$\underline{G} = - (\mu/R_C^3) \underline{R} \text{ where } R_C \text{ is a}$$

constant radius magnitude chosen judiciously. It is seen that

$\mu/R_C^3 = V_C^2/R_C^2 \equiv \omega^2$ where ω is the angular rotation rate of a circular orbit at radius R_C . This leads to

$$\underline{G} = - \omega^2 \underline{R}.$$

It will be seen in the following formulation that this approximation eliminates flat earth assumptions in the guidance law. The augmented functional becomes:

$$F = \underline{\lambda} \cdot [\dot{\underline{V}} - a_T \underline{e} + \omega^2 \underline{R}] + \underline{\alpha} \cdot (\dot{\underline{R}} - \underline{V}) + \beta (\dot{m} - f_7)$$

where for constant thrust $\dot{m} = f_7 = \text{constant}$, and for constant acceleration $f_7 = -ma_C/I_{sp}$ where a_C is the value of acceleration and I_{sp} is engine/propellant specific impulse (assumed constant in this problem), and

$$(\underline{\lambda} : \underline{\alpha} : \beta) \equiv \lambda_j, (\dot{\underline{V}} : \dot{\underline{R}} : \dot{m}) \equiv \dot{S}_j \text{ and } (\underline{V} : \underline{R} : m : \theta : \psi) \equiv S_i, j = 1, \dots, 7 \quad i = 1, \dots, 9$$

The typical powered flight trajectory and guidance problem* is to determine the thrust direction, $\underline{p}(t)$, time history that satisfies final conditions, $C_{j2} (\underline{V}_2 : \underline{R}_2)$, and simultaneously maximized final mass, m_2 . Therefore, $G \equiv m$. The end conditions, in general, are not explicit functions of t , m , and \underline{p} . The number of end conditions, s , can be anything from one to seven. In general, the initial conditions are fixed.

Applying equation (1)

$$(a) \quad \underline{\dot{\alpha}} = - \underline{\dot{\lambda}}$$

$$(b) \quad \underline{\dot{\alpha}} = \omega^2 \underline{\lambda} \rightarrow \underline{\lambda} + \omega^2 \underline{\lambda} = 0$$

$$(c) \quad \dot{\beta} = -(\underline{\lambda} \cdot \underline{p}) \frac{\partial a_T}{\partial m} - \beta \frac{\partial f_7}{\partial m}$$

$$(d) \quad \underline{\lambda} \cdot \left(\frac{\partial \underline{p}}{\partial \theta} \right) = \underline{\lambda} \cdot \left(\frac{\partial \underline{p}}{\partial \psi} \right) = 0 \rightarrow \underline{p} = \underline{\lambda} / |\underline{\lambda}|$$

* The problem here is restricted to a single burn maneuver, i.e., no long coast arcs between stages.

Referring to equation 3

(e) $H = C = \text{constant}$

since F is formally independent of time. Therefore, the term H disappears from the transversality condition for variable initial and final conditions; i.e.,

(f) $H_{t2} - H_{t1} = C - C = 0$

For simplicity it is assumed here that initial conditions are fixed and the initial differentials vanish, therefore, the transversality condition becomes

(g) $\beta_2 = -1$

(h) $\lambda_2 = - \sum_{j=1}^s \mu_j \frac{\partial C_{j2}}{\partial \underline{V}}, \alpha_2 = - \sum_{j=1}^s \mu_j \frac{\partial C_{j2}}{\partial \underline{R}} = - \dot{\lambda}_2$ since

$$dC_{j2} = \sum_{i=1}^n \frac{\partial C_{j2}}{\partial S_{i2}} dS_{i2} \text{ (end conditions are independent of } t, m, \theta \text{ and } \psi).$$

Conditions c, e, and g furnish superfluous information for the problem as stated, since they can be satisfied without controlling anything.

The solution to the differential equation of condition b (i.e., $\ddot{\lambda} - \omega^2 \lambda = 0$) is

$$\underline{\lambda} = \underline{\lambda}(0) \cos \omega t + \frac{\dot{\lambda}(0)}{\omega} \sin \omega t$$

$$\dot{\underline{\lambda}} = -\omega \underline{\lambda}(0) \sin \omega t + \dot{\underline{\lambda}}(0) \cos \omega t$$

In most low earth orbital maneuvers ω is approximately the range angle rate. Since this is a homogeneous set of equations (i.e., one of the components is arbitrary) the above equations can be expressed as

$$\underline{u} = \underline{\lambda} \cos \omega(t-K) + \frac{\dot{\underline{\lambda}}}{\omega} \sin \omega(t-K) = \underline{\lambda}(t) \quad (7)$$

$$\dot{\underline{u}} = -\omega \underline{\lambda} \sin \omega(t-K) + \dot{\underline{\lambda}} \cos \omega(t-K) = \dot{\underline{\lambda}}(t) \quad (8)$$

where t is referenced to $K = (1/2)t_2$, $t_1 = 0$, $\underline{\lambda} \equiv \underline{\lambda}(K)$ is a constant unit vector and $\dot{\underline{\lambda}} \equiv \dot{\underline{\lambda}}(K)$ is a constant rate vector. Equation 7 is the key to the simplicity and accuracy of this guidance algorithm, and replaces equation 4 of Reference 1. In general the quantity $\underline{\lambda} - \underline{\lambda}$ is small and $|\dot{\underline{\lambda}}| \approx \omega$, therefore \underline{u} is approximately a unit vector.

The thrust unit vector is expressed as

$$\underline{p} = \underline{u}/|\underline{u}| \quad (\text{condition d}) \quad (9)$$

Expanding equations 7 and 9 and dropping terms of order higher than three, \underline{p} can be approximated as

$$\underline{p} = \underline{\lambda} (1 - T\dot{x} - \frac{1}{2}\omega_1^2 x^2) + \dot{\underline{\lambda}} (x - T\dot{x}^2 - 1/6 \omega_2^2 x^3) \quad (10)$$

where $x = t-K$ and $T = \underline{\lambda} \cdot \dot{\underline{\lambda}}$, $T' = T (1 - 7/6 \omega_2^2 x^2)$, $\omega_1^2 = \dot{\underline{\lambda}} \cdot \dot{\underline{\lambda}}$, and $\omega_2^2 = 3\omega_1^2 - 2\dot{\omega}_1^2$.

The equation of motion becomes

$$\underline{a}_T \underline{p} = \underline{\dot{V}} - \underline{G} \quad (11)$$

The left hand side of equation 11 can be integrated twice in closed form using the generalized thrust integrals defined in Appendix A to yield

$$\underline{V}_{TH} \equiv \underline{L}_p \underline{\lambda} + \underline{J}_p \dot{\underline{\lambda}}$$

$$\underline{R}_{TH} \equiv \underline{S}_p \underline{\lambda} + \underline{Q}_p \dot{\underline{\lambda}} \quad \text{where}$$

$$\underline{L}_p = \underline{L} - T\underline{J}_1' - 1/2 \omega_1^2 \underline{J}_2, \quad \text{where } \underline{J}_1' = \underline{J}_1 - 7/6 \omega_2^2 \underline{J}_3$$

$$\underline{S}_p = \underline{S} - T\underline{Q}_1' - 1/2 \omega_2^2 \underline{Q}_2, \quad \text{where } \underline{Q}_1' = \underline{Q}_1 - 7/6 \omega_2^2 \underline{Q}_3$$

$$\underline{J}_p = \underline{J}_1 - T\underline{J}_2 - 1/6 \omega_2^2 \underline{J}_3$$

$$\underline{Q}_p = \underline{Q}_1 - T\underline{Q}_2 - 1/6 \omega_2^2 \underline{Q}_3$$

Equation 10 is probably adequate for all Shuttle maneuvers. However, if more accuracy is desired, a constant factor, f , can be introduced into equation 10, as in Reference 8. Then the average value of \underline{p} is forced to unity, i.e.:

$$\frac{1}{2K} \int_{-K}^K \underline{p} \cdot \underline{p} \, dx = 1$$

The above equation is a quadratic function of f , i.e., of the form

$$af^2 + bf + c = 0.$$

An alternate approach is to use equations 7 and 9 and employ numerical integration to integrate $(\underline{a}_T/|\underline{u}|)\cos \omega x$ and $(\underline{a}_T/|\underline{u}|)\sin \omega x$.

The right side of equation 11 can be handled as:

$$\int_0^{T_{GO}} (\dot{\underline{\lambda}} - \underline{G}) dt = \underline{V}_T - \underline{V} - \underline{V}_g \equiv \underline{V}_{GN}$$

$$\int_0^{T_{GO}} \int_0^t (\dot{\underline{\lambda}} - \underline{G}) dt^2 = \underline{R}_T - \underline{R} - \underline{V} T_{GO} - \underline{R}_g \equiv \underline{R}_{GN}$$

where \underline{V}_T and \underline{R}_T are terminal values of velocity and position. Gravity integrals (\underline{V}_g and \underline{R}_g) are defined in Appendix B. The velocity and position equations are expressed as two linear equations in $\underline{\lambda}$ and $\dot{\underline{\lambda}}$.

$$L_p \underline{\lambda} + J_p \dot{\underline{\lambda}} = \underline{V}_{GN} \quad (12)$$

$$S_p \underline{\lambda} + Q_p \dot{\underline{\lambda}} = \underline{R}_{GN} \quad (13)$$

Solving equations 12 and 13 simultaneously and forcing $\underline{\lambda}$ to unity it follows that

$$\underline{\lambda} = \text{Unit} (\underline{V}_{GN} - \underline{R}_{GN} J_p / Q_p) \equiv \text{unit} (\underline{V}_{GO}) \quad (14)$$

A scalar equation used to force \underline{R}_{GN} (i.e. \underline{R}_T) to be consistent with T_{GO} (time-to-go) is obtained by performing the dot product of $\underline{\lambda}$ with equation 13.

$$S_p + Q_p T = \underline{\lambda} \cdot \underline{R}_{GN}, \text{ where } T = \underline{\lambda} \cdot \dot{\underline{\lambda}}$$

This constraint can be satisfied by defining \underline{R}_{GN} as

$$\underline{R}'_{GN} = \left[\frac{S_p + Q_p T}{\underline{\lambda} \cdot \underline{R}_{GN}} \right] (\underline{R}_{GN}) \quad (15)$$

from which it follows that $\underline{\lambda} \cdot \underline{R}'_{GN} = S_p + Q_p T$. *

Substituting the right side of equation 15 into equation 13 (for \underline{R}_{GN})

$$\dot{\underline{\lambda}} = (S_p / Q_p) [(\underline{R}_{GN} / \underline{\lambda} \cdot \underline{R}_{GN}) - \underline{\lambda}] + T (\underline{R}_{GN} / \underline{\lambda} \cdot \underline{R}_{GN}) \quad (16)$$

*This same approach could be used in equation 12, possibly speeding convergence. First guesses for \underline{V}_{GN} and \underline{R}_{GN} could have any magnitude as long as the directions were somewhat reasonable.

Equation 16 can be bypassed if there are no position constraints. Inspection of equation 16 shows that $|\underline{R}_{GN}/\underline{\lambda} \cdot \underline{R}_{GN}|$ is the secant of the angle between \underline{R}_{GN} and $\underline{\lambda}$ (i.e., \underline{V}_{GO}) and that $(\underline{R}_{GN}/\underline{\lambda} \cdot \underline{R}_{GN}) - \underline{\lambda}$ is normal to $\underline{\lambda}$ and has the magnitude of the tangent of the same angle. So $\underline{\lambda}$ has a component normal to $\underline{\lambda}$ and a component along \underline{R}_{GN} equal to the product of $T = \underline{\lambda} \cdot \underline{\lambda}$ and the secant of the angle between \underline{R}_{GN} and \underline{V}_{GO} .

We now have values of $\underline{\lambda}$ and $\underline{\lambda}$, therefore the value of \underline{V}_{GO} can be defined for the next guidance cycle (from equation 12)

$$\underline{V}_{GO} = \underline{V}_{GN} - J_p \underline{\lambda} + (L - L_p) \underline{\lambda} \quad (17)$$

Time-to-go (T_{GO}) is determined from

$$L = |\underline{V}_{GO}|$$

Referring to the definitions of \underline{V}_{GN} , \underline{R}_{GN} , and Equations 12 and 13, the predicted values of \underline{V}_T and \underline{R}_T are obtained as:

$$\underline{V}_{PRED} = L_p \underline{\lambda} + J_p \underline{\lambda} + \underline{V} + \underline{V}_g \rightarrow \underline{V}_T \quad (18)$$

$$\underline{R}_{PRED} = S_p \underline{\lambda} + Q_p \underline{\lambda} + \underline{R} + \underline{V} T_{GO} + \underline{R}_g \rightarrow \underline{R}_T \quad (19)$$

quantities to be used in the end-condition routine in a predictor/corrector manner.

Having determined $\underline{p}(t)$ it is possible to employ numerical integration to solve equation 11 yielding any desired order of accuracy for state extrapolation and gravity effects. This would eliminate the generalized integrals of Appendix A and the need of an approximate gravity model; possibly simplifying the algorithm. However, this is not necessarily recommended. It would possibly result in an increased cycle time.

* e.g., the model of Appendix B.

3.1 Transversality Conditions

The following set of end conditions is defined to be used in development of the transversality conditions:

$$E_1 = V^2 - 2\mu/R \quad (\text{Energy}), V = |\underline{V}|, R = |\underline{R}|, \text{ and } V^2 = \underline{V} \cdot \underline{V}.$$

$$E_2 = (RV)^2 - (\underline{R} \cdot \underline{V})^2 \quad (\text{Angular momentum magnitude}) \quad (20)$$

$$E_3 = (\underline{R} \times \underline{V}) \cdot \underline{U}_y \quad (\text{Angular momentum vector})$$

Where \underline{R} is radius vector, \underline{V} is velocity vector, μ is the gravity constant and \underline{U}_y is the desired value of the angular momentum unit vector. These conditions are to be satisfied at the end of a maneuver. For convenience, final subscripts are dropped. The general transversality condition from the last section is:

$$\underline{u} = - \sum_{j=1}^s \mu_j \frac{\partial C_j}{\partial \underline{V}} \quad (21)$$

$$\dot{\underline{u}} = \sum_{j=1}^s \mu_j \frac{\partial C_j}{\partial \underline{R}}$$

Where s is the number of end conditions (condition h).

Case 1

Consider the case where energy is the only end condition (i.e., $s=1$, $C_1=E_1$). Using equations 20 and 21

$$\underline{u} = -2\mu \underline{V} \quad (22)$$

$$\dot{\underline{u}} = -2\mu \underline{G}$$

Where $\underline{G} = - \frac{\mu}{R^3} \underline{R}$ is the gravity vector.

Equation 22 implies that the final value of \underline{u} is parallel to the velocity vector and the final value of $\dot{\underline{u}}$ is parallel to the gravity vector.

Case 2

Consider the case of two end conditions, energy and angular momentum.

$$s = 2, C_1 = E_1, C_2 = E_2$$

Using equations 20 and 21

$$-\dot{\underline{u}} + \mu_1 \underline{N}_V - \mu_2 \underline{G} = 0$$

$$-\underline{u} - \mu_1 \underline{N}_R + \mu_2 \underline{V} = 0$$

Where $\underline{N}_V = \underline{V} \times (\underline{R} \times \underline{V})$ and $\underline{N}_R = \underline{R} \times (\underline{R} \times \underline{V})$.

Performing the appropriate dot products the following scalar equations result:

$$-\dot{\underline{u}} \cdot \underline{V} - \mu_2 \underline{G} \cdot \underline{V} = 0$$

$$\underline{u} \cdot \underline{G} + \mu_2 \underline{G} \cdot \underline{V} = 0$$

$$-\dot{\underline{u}} \cdot \underline{u} + \mu_1 \underline{u} \cdot \underline{N}_V - \mu_2 \underline{u} \cdot \underline{G} = 0$$

$$\underline{u} \cdot \underline{u} - \mu_1 \dot{\underline{u}} \cdot \underline{N}_R + \mu_2 \dot{\underline{u}} \cdot \underline{V} = 0$$

from which the two necessary conditions result

$$\dot{\underline{u}} \cdot \underline{V} - \underline{u} \cdot \underline{G} = 0 \quad (23)$$

$$\dot{\underline{u}} \cdot \underline{N}_R - \underline{u} \cdot \underline{N}_V = 0 \quad (24)$$

Equations 23 and 24 can be written as

$$\dot{\underline{u}} = (\underline{u} \cdot \underline{G} / \dot{\underline{u}} \cdot \underline{V}) \dot{\underline{u}} \quad (23)$$

$$S_{\theta} = - (V/R\dot{u}_m) u_m S_{\alpha} \quad (24)$$

where $\dot{\underline{u}}_u = \text{Unit}(\dot{\underline{u}})$, $\dot{u}_m = |\dot{\underline{u}}|$, $S_{\theta} \equiv \sin(\dot{\underline{u}}, \underline{G})$,

$u_m = |\underline{u}|$, $\underline{u}_p = \text{unit}(\underline{N}_V)$, and

$S_{\alpha} = [\text{unit}(\underline{u})] \cdot \underline{u}_p = \sin(\text{angle of attack } -\alpha)$.

Inspecting equation 24 it is seen that the turning rate vector is rotated from the terminal gravity vector (negative radius vector) by an angle (θ) that is proportional to the terminal angle of attack. In many maneuvers

$(V/R\dot{u}_m) u_m \approx 1$, when $\frac{V}{R} \approx \omega$ (angular rotation rate),

$\dot{u}_m \approx \frac{V}{R}$, and $u_m \approx 1$. In this case

$S_{\theta} \approx S_{\alpha}$ or $\theta \approx \alpha$.

Equation 23 provides magnitude information and equation 24, direction information.

Case 3

Consider the case of three constraints

$$s = 3, C_1 = E_1, C_2 = E_2, C_3 = E_3$$

Using equations 20 and 21 and performing the necessary algebraic manipulation, it is seen that equations 23 and 24 are the necessary conditions for optimality as in Case 2. It can also be shown that these are the transversality conditions for a velocity vector constraint (i.e., V, γ, A_z) with no position constraint. E_3 can be considered an azimuth constraint rather than a position constraint.

Case 4

Considering the case involving position constraint(s) as well as velocity constraint(s), it can be shown that equation 24 is the transversality condition and equation 23 is not used.

In summary, three modes suffice to handle all sets of end-conditions (i.e., end of thrusting maneuver constraints):

- Mode 1 - Energy constraint,
- Mode 2 - Velocity vector constraints, no position constraint(s), and
- Mode 3 - Position constraint(s).

The transversality conditions are implemented in the LTG algorithm with very simple modifications. Terminal values of \underline{u} and $\dot{\underline{u}}$ require several pre-guidance iterations as do T_{GO} , \underline{V}_T , and \underline{R}_T (time-to-go, terminal velocity and position vectors). However, after several iterations all the terminal conditions converge to virtually constant values (i.e., to within the accuracy of the guidance algorithm). The following definitions are made:

$$R_D = |\underline{R}_T|$$

$$\underline{U}_y = \text{Unit vector normal to desired orbit plane, input or calculated inflight.}$$

$$\underline{U}_x = \text{unit } (\underline{R}_T)$$

$$\underline{U}_z = \underline{U}_x \times \underline{U}_y$$

All of the above defined quantities are defined and available in the LTG algorithm. An arbitrary manner of implementing the transversality conditions is shown here. For convenience, final subscripts are dropped. From equations 7 and 8 where $t = 2K$:

$$\underline{u} = \underline{\lambda} \cos \omega K + (\dot{\underline{\lambda}}/\omega) \sin \omega K \quad (7-a)$$

$$\dot{\underline{u}} = -\omega \underline{\lambda} \sin \omega K + \dot{\underline{\lambda}} \cos \omega K \quad (8-a)$$

The various modes are handled as follows:*

MODE 1 - Energy

$$\underline{u}_y = \text{unit } (\underline{V})$$

$$\underline{u} = (\underline{u} \cdot \underline{u}_y) \underline{u}_y$$

$$\dot{\underline{u}} = (\dot{\underline{u}} \cdot \underline{u}_x) \underline{u}_x$$

*A reasonable first guess estimate for $\dot{\underline{\lambda}}$ (for all modes) results from equation 7-a assuming that $\underline{u} = \text{unit}(\underline{V})$, i.e., final angle of attack is small.

MODE 2 - Velocity

$$\dot{\underline{u}}_u = \text{unit}(\dot{\underline{u}})$$

$$\dot{\underline{u}} = (\underline{u} \cdot \underline{G} / \dot{\underline{u}}_u \cdot \underline{V}) \dot{\underline{u}}_u \text{ or } \dot{\underline{\lambda}}_u = \text{unit}(\dot{\underline{\lambda}}) \text{ and}$$

$$\dot{\underline{\lambda}} = \frac{\dot{\underline{\lambda}}_u \cdot (\underline{G} \cos \omega K + \underline{V} \omega \sin \omega K)}{\dot{\underline{\lambda}}_u \cdot [\underline{V} \cos \omega K - (\underline{G}/\omega) \sin \omega K]} \dot{\underline{\lambda}}_u$$

$$\dot{\underline{u}} = -\omega \underline{\lambda} \sin \omega K + \dot{\underline{\lambda}} \cos \omega K$$

MODES 2 and 3 - Velocity or Position

$$\dot{\underline{u}}_m = |\dot{\underline{u}}|$$

$$S_e = [\text{Unit}(\dot{\underline{u}})] \cdot \underline{u}_z + \underline{u} \cdot (\underline{u}_y \times \underline{V}) / \dot{\underline{u}}_m R_D \rightarrow 0$$

$$\dot{\underline{u}} = \dot{\underline{u}} + S_e [\dot{\underline{u}}_m \underline{u}_z - (\dot{\underline{u}} \cdot \underline{u}_z) \dot{\underline{u}}] \text{ (small angle approximation)}$$

and finally, $\dot{\underline{\lambda}}$ is obtained for all modes by solving equations 7-a and 7-b simultaneously

$$\dot{\underline{\lambda}} = \dot{\underline{u}} \cos \omega K + \omega \underline{u} \sin \omega K \quad (25)$$

The quantity $\dot{\underline{\lambda}}$ is practically a function of $K = (1/2) T_{GO}$ only, since $\dot{\underline{u}}$ and \underline{u} converge to virtually constant values after a few passes through the guidance algorithm.

Equation 25 should be evaluated before and after the transversality computation.

4.0 END-CONDITIONS ..

It is simple to implement this guidance algorithm to handle any reasonable set of end conditions in an optimum fashion. For example let the desired end conditions be velocity magnitude (V_D), radius magnitude (R_D), flight path angle (γ_D), and orbital inclination (i_D). i.e., there are four end conditions and the downrange angle and nodal angle are free. A position constraint (R_D) is involved and it will be seen that the transversality condition is handled as in mode 3 in paragraph 3.1. A simple iterative manner of handling this problem is as follows:

Using equations 18 and 19:

$$\underline{V} = V_D \text{ Unit } (\underline{V}_{\text{PRED}}), \quad \underline{R} = R_D \text{ Unit } (\underline{R}_{\text{PRED}})$$

$$\underline{U}_x = \text{Unit } (\underline{R}), \quad \underline{U}_y = \text{Unit } (\underline{V} \times \underline{U}_x), \quad \underline{U}_z = \underline{U}_x \times \underline{U}_y$$

Define an error term:

$S_e = a_1 (\underline{\Omega} \cdot \underline{U}_y + \cos i_D)$ where a_1 is a damping coefficient ($a_1 \leq 1/\cos \psi$)* empirically chosen, and $\underline{\Omega}$ is the earth rotation unit vector.

Then the solution proceeds as follows:

$$C_e = \sqrt{1 - S_e^2}$$

$$\underline{V}_T = V_D \begin{bmatrix} \underline{U}_x \\ \underline{U}_y \\ \underline{U}_z \end{bmatrix}^T \begin{bmatrix} \sin \gamma_D C_e \\ S_e \\ \cos \gamma_D C_e \end{bmatrix}, \quad \underline{R}_T = R_D \text{ Unit } (\underline{R} + S_p S_e \underline{U}_y)$$

When $S_e \rightarrow 0$ end conditions and the transversality condition are satisfied.

Another example is in Reference 9 where the magnitude of apogee radius (R_a) is the only constraint. The transversality condition is Mode 1-energy. This end-condition scheme makes use of the relationship

$R_p = [R_D(R_a - R_D)] / [(R_a / \cos^2 \gamma_D) - R_D]$, where R_p is perigee radius.** Another relationship that could be used to develop end-condition schemes is

$$R_p [2 - (1 - \cos \theta)(R_D / R_a)] - R_D (1 + \cos \theta) = 0, \text{ where } \theta \text{ is true anomaly.}$$

* Where ψ is terminal latitude.

** R_D and γ_D are computed from predicted values, $\underline{R}_{\text{PRED}}$ and $\underline{V}_{\text{PRED}}$.

4.1 Ascent Orbiter Maneuvering System (OMS) Burn (one constraint-Apogee Control)

A present ground rule for OMS ascent burn is to have a separate OMS guidance and targeting stage. During main engine burn guidance is targeted explicitly for Main Engine Cutoff (MECO) conditions to accommodate external tank (ET) impact. After MECO guidance is restarted and the OMS is targeted for different conditions. There is a very low acceleration during OMS burn ($\approx .06$ g's); therefore velocity direction change and altitude constraints could be very expensive in terms of performance and guidance and control (G&C) systems stability. (e.g., a velocity change of one degree at a velocity of 25,000 FPS costs about 450 FPS, compared to a total nominal OMS propellant loading of 600 FPS. An altitude constraint could also be very expensive and produce erratic attitude and attitude rate commands).

The guidance and targeting scheme, presented herein for implementation in the SSFS, produces an optimum (minimum ΔV) OMS burn to achieve a desired target apogee altitude (or radius). Perigee altitude is not explicitly constrained but is implicit in terms of desired apogee altitude and OMS cutoff altitude and flight path angle. Predicted values of cutoff altitude and flight path angle are obtained by enforcing a near gravity turn trajectory for the OMS burn, until a velocity magnitude is attained that satisfies apogee altitude. The problem is to determine the desired velocity magnitude as a function of desired apogee altitude and predicted cutoff altitude and flight path angle.

The three following orbital mechanics equations are used to express perigee radius (R_p) as a function of desired apogee radius (R_A), burnout radius (R_D), and burnout flight path angle (γ_D).

$$a = .5 (R_A + R_p) \quad (26)$$

$$e = (R_A - R_p) / (R_A + R_p) \quad (27)$$

$$\cos^2 \gamma_D = [a^2(1-e^2) / R(2a-R)] \quad (28)$$

Substituting equations 26 and 27 into equation 28 results in

$$R_p = [R_D(R_D - R_A) \cos^2 \gamma_D / (R_D \cos^2 \gamma_D - R_A)] .$$

Then desired velocity is as follows

$$V_D = \left[\mu \left(\frac{2}{R_D} - \frac{1}{a} \right) \right]^{1/2} \text{ or}$$

$$V_D = [2\mu R_A(R_D - R_A) / R_D(R_D^2 \cos^2 \gamma_D - R_A^2)]^{1/2} \quad (29)$$

The value of $\cos^2 \gamma_D$ is obtained from values of cutoff position and velocity vectors (\underline{R}_T and \underline{V}_T).

$$\cos^2 \gamma_D = 1 - [\text{Unit}(\underline{R}_T) \cdot \text{Unit}(\underline{V}_T)]^2 \quad (30)$$

R_D is expressed as

$$R_D = |\underline{R}_T| \quad (31)$$

This guidance option is implemented in the following order:

- (1) R_D is obtained from equation 31,
- (2) $\cos^2 \gamma_D$ comes from equation 30,
- (3) V_D is obtained from equation 29, and
- (4) The desired velocity vector is expressed as $\underline{V}_T = V_D \text{Unit}(\underline{V}_T)$

In Section 3.1 it is shown that a necessary condition for optimality (for the one constraint problem) is that the final thrust direction be parallel to the final velocity vector (\underline{V}_T). This condition is satisfied by the following equation, (if the simplifying approximation that $\underline{\lambda} \cdot \underline{\lambda} = 0$ is made).

$$\dot{\underline{\lambda}} = (\omega / \tan \omega K) [(\underline{V}_T / \underline{\lambda} \cdot \underline{V}_T) - \underline{\lambda}]$$

where

$$\omega = \mu / VR^2$$

and V and R are initial values of velocity and radius magnitude.

This guidance option was described in Reference 9 and implemented in the SSFS. The option has also been implemented in the Unified Powered Flight Guidance of Reference 10.

An approximation of the above equation is

$$\dot{\underline{\lambda}} = -\omega \text{unit}[(\underline{R}_{GN} / \underline{\lambda} \cdot \underline{R}_{GN}) - \underline{\lambda}],$$

which is essentially what is implemented in Reference 10.

4.2 Range Throttling

For Shuttle mission 3-B, it is a requirement to determine a constant thrust level to achieve a prescribed reference trajectory (for rendezvous purposes). Also, for Return to Launch Site (RTLS) abort, it is desirable to throttle to a constant thrust level to achieve a prescribed range from the landing site at Main Engine Cutoff (MECO).

Equation 13 of Section 3.0 is used to develop a throttle equation.

$$S_p \dot{\lambda} + Q_p \dot{\lambda} = \underline{R}_{GN} \equiv \underline{R}_T - \underline{R} - \underline{V} T_{GO} - \underline{R}_g \quad (13)$$

where \underline{R}_T is the fixed, desired terminal radius vector.

A description of the method is simplified by making the assumption that $T \equiv \lambda \cdot \dot{\lambda} = 0$. Performing the vector scalar product of $\underline{\lambda}$ with equation 13 results in

$$S_p = \underline{\lambda} \cdot \underline{R}_{GN} \quad (32)$$

In order to satisfy equation 32, it is necessary to know the proper value of thrust, mass flow rate, and T_{GO} (time-to-go). Thrust (F) and mass flow rate (\dot{m}) are constant and directly proportional to each other, i.e.,

$$F/\dot{m} = V_{ex} \text{ (constant exhaust velocity).}$$

T_{GO} is inversely proportional to thrust, or visa versa.

The authors of Reference 10 developed the concept of analytically determining the time-to-go (T_{GO}) to satisfy a range equation (similar to equation 32). They then used this new value of T_{GO} (along with the old value, T_{GO}^i) to define a proportionality constant (K) relating desired values of F and \dot{m} to prescribed reference values (F_R and \dot{m}_R), i.e.

$$F = K F_R \text{ and } \dot{m} = K \dot{m}_R \quad (33)$$

The value of K is initialized as $K' = 1.00$ and on subsequent iterations is altered as

$$K = K' T_B^i / T_B \text{ and } K' \text{ is reset as } K' = K, \quad (34)$$

where T_B is the burn time until an acceleration limit is attained, or $T_B = T_{GO}$ if an acceleration limit will not be attained. T_B^i is the value of T_B on the previous iteration and $T_B = T_B^i + \Delta T_{GO}$, where ΔT_{GO} is the desired increment in T_{GO} to satisfy the range equation. The value of ΔT_{GO} is obtained from a Newton-Raphson iteration employed in satisfying the range equation. Each guidance pass represents one iteration, however, the solution is virtually converged after a few passes.

The approach described in the above paragraph is taken here; however, the solution for ΔT_{GO} to satisfy the range constraint is approached in a different manner. A value of $T_{GO} = T'_{GO} + \Delta T_{GO}$ is found to satisfy equation 32, i.e.,

$$S_p = \lambda \cdot R_{GN} \text{ or } \epsilon = S_p - (\lambda \cdot R_{GN}) \rightarrow 0, \quad (35)$$

where ϵ is an error to be driven to zero.

For small values of ΔT_{GO} , a very good approximation for S_p is

$$S_p = S'_p + .5L_p (T_{GO} - T'_{GO}) \text{ where } S'_p \text{ is the uncorrected value.}$$

Another good assumption for variable thrust is

$$\frac{\partial L_p}{\partial T_{GO}} = 0, \text{ since } L_p \text{ is essentially an average acceleration multiplied}$$

by T_{GO} , and if the acceleration is changed by a given factor, T_{GO} is changed inversely by the same factor.

Therefore a good approximation is

$$\frac{\partial S_p}{\partial T_{GO}} = .5L_p.$$

Another approximation used here is

$$\frac{\partial R_g}{\partial T_{GO}} = \underline{V}_g, \text{ where } \underline{R}_g \text{ and } \underline{V}_g \text{ are gravity integrals.}$$

Equation 35 is expressed as

$$\epsilon = S_p - \lambda \cdot (\underline{R}_T - \underline{R} - \underline{V}T_{GO} - \underline{R}_g). \quad (35)$$

Differentiation of equation 35 yields

$$\frac{\partial \epsilon}{\partial T_{GO}} = .5L_p - \lambda \cdot (-\underline{V} - \underline{V}_g). \quad (36)$$

Adding $\lambda \cdot \underline{V}_T$ to equation 36 and subtracting the same quantity yields

$$\frac{\partial \epsilon}{\partial T_{GO}} = .5L_p - \lambda \cdot (\underline{V}_T - \underline{V} - \underline{V}_g) + \lambda \cdot \underline{V}_T$$

Using the identity .

$\underline{V}_T - \underline{V} - \underline{V}_g = \underline{V}_{GN} = L_p$, it follows that

$$\frac{\partial \epsilon}{\partial T_{GO}} = \underline{\lambda} \cdot \underline{V}_T - .5 L_p . \quad (36)$$

And finally,

$$\Delta T_{GO} = -\epsilon / \frac{\partial \epsilon}{\partial T_{GO}} , \quad (37)$$

Where ϵ is defined by equation 35 and $\frac{\partial \epsilon}{\partial T_{GO}}$, by equation 36.

Equation 34 is used to define K (where $T_B = T_B' + \Delta T_{GO}$) and the desired value of F and \dot{m} are obtained from equation 33.

The only modifications necessary to implement this method into the range throttling routine of Reference 10 is to make the following definitions.

$$V_{goz} = L_p$$

$$\Delta r_{\Delta tgo} = \underline{\lambda} \cdot \underline{V}_T - .5 V_{goz} = \partial \epsilon / \partial T_{GO}$$

$$\Delta r_z = \underline{\lambda} \cdot (\underline{R}_T - \underline{R} - \underline{V} T_{GO} - R_g) - S_p = -\epsilon$$

5.0 RETURN TO LAUNCH SITE ABORT (PROPELLANT DEPLETION CONSTRAINT)

The objective of Return to Launch Site (RTLS) abort guidance is to provide closed loop pitch and yaw commands to achieve explicit burnout conditions at External Tank (ET) fuel depletion. The explicit conditions at Main Engine Cutoff (MECO) are relative velocity (V_R), relative flight path angle (γ_R), altitude (h), relative flight azimuth (A_{ZR}), range from landing site (R_{LS}) and burnout weight (W_{BO}). A 3G acceleration limit is also imposed. The RTLS targeting algorithm is not presented in this report since it is presented as one of the targeting options in Figure 5-11 of Reference 10. Equations to constrain vehicle attitude at MECO are easily developed, should this become a requirement.

This Section presents two powered flight guidance algorithms capable of handling the burn to MECO portion of Shuttle RTLS abort. Throttling capability is provided to constrain range from the landing site at MECO. However, throttling is not necessary should it be desirable to relax the range constraint. The work presented in Section 5.1 is based on the concept developed by Russ Sievers (Reference 11), called Launch Abort Guidance Simulation (LAGS). LAGS is presently being used in the Space Vehicle Dynamic Simulator (SVDS) for RTLS performance and targeting studies, and is currently being programmed in the SSFS.

The LAGS guidance concept is to have a constant attitude maneuver before the nominal guidance maneuver, and the value of the constant attitude is computed explicitly, simultaneously with the parameters of the nominal guidance phase (i.e., the constant attitude phase is not open-loop). The duration time of the constant attitude phase is proportional to the excess velocity or excess propellant. Time-to-go is an explicit function of desired burnout weight.

The guidance algorithm presented in Section 5.2 requires no constant attitude maneuver to expend excess propellant. The algorithm represents an explicit solution to the seven constraint guidance problem (with constant thrust level before acceleration limit)--burnout weight is explicitly constrained as well as terminal position and velocity vectors. Burnout weight is controlled by burn time, the steering modulates to expend excess propellant (while satisfying radial and crossrange position and velocity constraints), and a constant thrust level (before acceleration limit) is determined that satisfies the downrange constraint.

The "single maneuver" algorithm of Section 5.2 has advantages over the "two maneuver" algorithm of Section 5.1. It is much simpler to implement as a guidance option in the nominal guidance (with the addition of a negligible amount of equations and logic). It also eliminates most of the disadvantages of other candidate powered flight RTLS guidance schemes (a comparison of candidate schemes is out of the scope of this report).

Atmospheric terms are not explicitly included; however, an empirical velocity bias term (that ranges from full value to zero at MECO) is introduced to insure that enough propellant is available to achieve desired velocity and burnout weight simultaneously. This bias term also compensates for other approximations in the guidance algorithm. Time-to-go (T_{GO}) is computed explicitly as a function of nominal desired burnout weight. If more propellant is available than required to achieve the velocity, the steering modulates to deplete the excess propellant.

5.1 Derivation of Constant Attitude Equations

The RTLS guidance equations will be developed here in the order of computation. The following definitions are made:

V_{ex} - Engine exhaust velocity

a - Measured acceleration

a_L - Acceleration limit (e.g., 3 g's)

$\tau = V_{ex}/a$

W - Current vehicle weight (onboard estimation)

W_{BO} - Nominal MECO burnout weight (input)

Time-to-go is computed as follows:

$T_P = \tau(1-a/a_L)$ - (Time of constant thrust)

$T_{LS} = (V_{ex}/a_L) \ln (W/a_L W_{BO})$ (Time of constant acceleration)

$T_{GO} = T_P + T_{LS}$ (Time-to-go)

Additional parameters are computed as follows:

Velocity available:

$$V_a = V_{ex} \ln [\tau/(\tau-T_P)] + a_L T_{LS} - L_\theta$$

where L_θ is the velocity increment during the reorientation maneuver (computed later in the order of computation). L_θ is initialized as zero on the first pass. Velocity required is updated as:

$V_{REQ} = V_{REQ} - \Delta V_{sensed}$ where ΔV_{sensed} is the measured velocity increment since the last guidance pass. The excess velocity is now computed.

$L_c = V_a - |V_{REQ}|$ and the time of the constant attitude maneuver is computed as $T_1 = \tau [1 - \exp (-L_c/V_{ex})]$.

The equations of motion assume the following form:

$$\dot{\underline{V}}_1 = a_1 \underline{C}_1 + \underline{G}, \quad 0 \leq t \leq T_1 \quad (t \text{ is time})$$

$$\dot{\underline{V}}_2 = a_2 \left\{ [1 - 1/2\omega^2(t-K)^2] \underline{\lambda} + [(t-K) - 1/6\omega^2(t-K)^3] \dot{\underline{\lambda}} \right\} + \underline{G}$$

$$0 \leq t \leq T_2 \text{ where } T_2 = T_{Go} - T_1.$$

Where

$$a_1 = (F/m)_1$$

$$a_2 = (F/m)_2$$

F - Thrust (function of time)

m - Vehicle mass (function of time)

\underline{C}_1 - Unit vector defining constant attitude phase

\underline{G} - Gravity (function of radius vector)

$\underline{\lambda}$ - Unit vector defining mid-point of nominal guidance phase

$\dot{\underline{\lambda}}$ - Pitch rate vector during nominal guidance phase

For simplicity the following constraints are introduced into the above equations

$$\underline{\lambda} \cdot \dot{\underline{\lambda}} = 0$$

$$\omega^2 = \dot{\underline{\lambda}} \cdot \dot{\underline{\lambda}}$$

The expression for $\dot{\underline{V}}_2$ was derived in Section 3.0 (assuming that $\underline{\lambda} \cdot \dot{\underline{\lambda}} = 0$).

The following integrals are defined:

$$\int_0^{T_1} a_1 dt = L_c \text{ (excess velocity)}$$

$$S_c = \int_0^{T_1} \int_0^t a_1 dt dt = (T_1 - \tau) L_c + V_{ex} T_1$$

Where $a_1 = V_{ex}/(\tau - t)$

The following integrals are defined in Appendix A.

$$L = \int_0^T a_2 dt$$

$$S = \int_0^T \int_0^t a_2 dt dt$$

$$J_m = \int_0^T a_2 (t-K)^m dt$$

$$Q_m = \int_0^T \int_0^t a_2 (t-K)^m dt dt, m=1, \dots, 3$$

Where $\tau_2 = \tau - T_1$ and $a_2 = v_{ex}(\tau_2 - t)$ or $a_2 = a_L$

$$K = 1/2(T_{Go} - T_1).$$

Gravity integrals are defined in References 2 and 3 and Appendix B.

$$\underline{V}_g = \int_0^T \underline{G} dt$$

$$\underline{R}_g = \int_0^T \int_0^t \underline{G} dt dt.$$

Using the above integrals the equations of motion are integrated twice to yield

$$L_{C-1} + L_p \lambda = \underline{V}_T - \underline{V} - \underline{V}_g - J_p \dot{\lambda} \equiv \underline{V}_{GN}$$

$$S_{C-1} + S_p \lambda + Q_p \dot{\lambda} + L_{C-1} T_2 = \underline{R}_T - \underline{R} - \underline{V} T_{Go} - \underline{R}_g \equiv \underline{R}_{GN}$$

Where

$$L_p = L - 1/2 \omega^2 J_2$$

$$S_p = S - 1/2 \omega^2 Q_2$$

$$J_p = J_1 - 1/6 \omega^2 J_3$$

$$Q_p = Q_1 - 1/6 \omega^2 Q_3$$

\underline{V}_T is terminal velocity vector (at MECO)

\underline{R}_T is terminal radius vector

\underline{R} is current radius vector

\underline{V} is current velocity vector

The integral S_c can be redefined as

$$S_c = (T_1 - \tau)L_c + V_{ex}T_1 + L_cT_2 = (T_{Go} - \tau)L_c + V_{ex}T_1$$

The above equations now assume the form

$$L_c \underline{C}_1 + L_p \underline{\lambda} = \underline{V}_{GN} \quad (38)$$

$$S_c \underline{C}_1 + S_p \underline{\lambda} + Q_p \dot{\underline{\lambda}} = f \underline{R}_{GN} \quad (39)$$

The unknown scalar factor f is introduced because it is assumed that the direction of \underline{R}_{GN} is known but the magnitude is an unknown function of the thrust integrals. The desired terminal radius magnitude (R_D) is an input, however, the down range component of the terminal radius vector (\underline{R}_T) is unknown. Even if throttling is used to constrain down range position, it is assumed to be unknown in solving equations 38 and 39. At another point in the equations, the thrust level is corrected to correct the difference between desired radius vector and predicted radius vector (based on equation 39).

Equations 38 and 39, in component form, represent six linear equations in ten unknowns (\underline{C}_1 , $\underline{\lambda}$, $\dot{\underline{\lambda}}$, and f). However, four scalar equations are introduced, making the solution possible in a recursive manner. Equation 38 is solved independent of equation 39 for \underline{C}_1 and $\underline{\lambda}$. Then values of \underline{C}_1 and $\underline{\lambda}$ are substituted into equation 39 from which $\dot{\underline{\lambda}}$ is solved. Performing the appropriate vector dot products the following scalar equations result.

$$L_c^2 + 2L_c L_p \underline{\lambda} \cdot \underline{C}_1 + L_p^2 = \underline{V}_{GN} \cdot \underline{V}_{GN} \equiv V_{GN}^2$$

$$L_c + L_p \underline{\lambda} \cdot \underline{C}_1 = \underline{C}_1 \cdot \underline{V}_{GN} \equiv V_{GN} C_{\theta}$$

$$L_c \underline{\lambda} \cdot \underline{C}_1 + L_p = \underline{\lambda} \cdot \underline{V}_{GN} \equiv V_{GN} C_{\lambda}$$

$$S_c \underline{\lambda} \cdot \underline{C}_1 + S_p = f \underline{\lambda} \cdot \underline{R}_{GN}$$

$$\text{since } \underline{C}_1 \cdot \underline{C}_1 = 1, \underline{\lambda} \cdot \underline{\lambda} = 1, \text{ and } \underline{\lambda} \cdot \dot{\underline{\lambda}} = 0.$$

Defining $C_\alpha = \underline{\lambda} \cdot \underline{\dot{C}}_1$ it follows that:

$$C_\alpha = 1 - [(L_c + L_p)^2 - v_{GN}^2] / 2L_c L_p \quad (40)$$

$$C_\theta = (L_c + L_p C_\alpha) / v_{GN} \quad (41)$$

$$C_\lambda = (L_c C_\alpha + L_p) / v_{GN} \quad (42)$$

$$f = (S_c C_\alpha + S_p) / (\underline{\lambda} \cdot \underline{R}_{GN}) \quad (43)$$

A solution to equation 38 is constructed by making the following assumptions:

$\underline{\dot{C}}_1$, $\underline{\lambda}$, and $\underline{\dot{\lambda}}$ all lie in the plane defined by

$\underline{U}_v = \text{Unit}(\underline{v}_{GN})$ and

$\underline{N} = \text{Unit}[(\underline{R}_{GN} / \underline{U}_v \cdot \underline{R}_{GN}) - \underline{U}_v]$.

Another assumption is that $\underline{\dot{C}}_1 \cdot \underline{N} > 0$ and $\underline{\lambda} \cdot \underline{N} < 0$. It is seen that C_α is the cosine of the angle between $\underline{\lambda}$ and $\underline{\dot{C}}_1$, C_θ is the cosine of the angle between $\underline{\dot{C}}_1$ and \underline{v}_{GN} , and C_λ is the cosine of the angle between $\underline{\lambda}$ and \underline{v}_{GN} .

The value of C_α is determined from equation 40 and substituted into equations 41 and 42 for C_θ and C_λ . Then the solution for $\underline{\dot{C}}_1$ and $\underline{\lambda}$ follows as:

$$\begin{aligned} \underline{\dot{C}}_1 &= C_\theta \underline{U}_v + (\sqrt{1 - C_\theta^2}) \underline{N} \\ \underline{\lambda} &= C_\lambda \underline{U}_v - (\sqrt{1 - C_\lambda^2}) \underline{N} \end{aligned} \quad (44)$$

The steering command during the constant attitude phase is

$$\underline{\dot{D}} = \underline{\dot{C}}_1. \quad (45)$$

Now that $\underline{\lambda}$ is known the value of f is determined from equation 43.

$$f = (S_c C_\alpha + S_p) / (\underline{\lambda} \cdot \underline{R}_{GN})$$

In solving equation 39 for $\underline{\dot{\lambda}}$ it is desirable to force the radial magnitude constraint, $|\underline{R}_T| = R_D$. This is done in the following manner:

$$\underline{R}_c = \underline{R} + \underline{V}T_{Go} + \underline{R}_g$$

$$\underline{R}_T = R_D \text{ Unit } (f\underline{R}_{GN} + \underline{R}_c)$$

$$\underline{R}_{GN} = \underline{R}_T - \underline{R}_c$$

$$f = (S_c C_\alpha + S_p) / (\lambda \cdot \underline{R}_{GN})$$

The solution for $\dot{\lambda}$ (from equation 39) follows as

$$\dot{\lambda} = (f\underline{R}_{GN} - S_c \underline{C}_1 - S_p \lambda) / Q_p$$

The following parameters are now computed for the next guidance pass:

$$\omega = |\dot{\lambda}|$$

(Initial guidance command of nominal guidance)

$$\underline{P}_0 = \lambda \cos \omega K - (\dot{\lambda} / \omega) \sin \omega K$$

$$C_{os} = \underline{P}_0 \cdot \underline{C}_1$$

(Unit vector halfway between \underline{P}_0 and \underline{C}_1 .)

$$\underline{C}_2 = (\underline{P}_0 + \underline{C}_1) / \sqrt{2(1 + C_{os})}$$

(Duration time of reorientation maneuver)

$$T_\theta = [\pi/2 - \sin^{-1}(C_{os})] / \omega_L$$

Where ω_L is pitch rate limit (e.g. 10°/sec)

(Velocity increment during reorientation phase)

$$L_\theta = V_{ex} \ln[(\tau - T_1) / (\tau - T_1 - T_\theta)]$$

(Velocity loss factor due to turning-approximate)

$$d = 1 - 1/6 \omega_L^2 (T_\theta/2)^2$$

The expression for d is exact assuming constant average acceleration ($a = L_\theta / T_\theta$)

$$dL_\theta = \int_0^{T_\theta} a [1 - 1/2 \omega_L^2 (t - T_\theta/2)^2] dt,$$

The velocity required is now computed for the next guidance pass.

$$\underline{V}_{Go} = \underline{V}_T - \underline{V} - \underline{V}_g - J_p \dot{\lambda} + (L - L_p) \underline{\lambda}$$

$$\underline{V}_{REQ} = \underline{V}_{Go} - dL_\theta \underline{C}_2 + \underline{V}_{BIAS}$$

where $\underline{V}_{BIAS} = B [T_{Go} / (T_{Go} + T - T_0)]^2 \text{ Unit } (\underline{V}_{Go})$. B is an empirical bias and T_0 is a reference time.

5.2 Derivation of Steering Modulation Equations (for fixed burnout weight)

In the following analysis, T_{GO} and the thrust integrals L , S , J_i , Q_i ($i = 1, \dots, 3$) are fixed (for a fixed thrust level), since the burnout weight is fixed. Therefore, the steering must modulate to burn up excess propellant, as well as satisfy position and velocity constraints. It is assumed that excess propellant is available, whether the amount is considerably large or approaches zero (the amount converges to virtually zero at thrust termination).

The following equations are developed in a second order manner (rather than third order) in order to be compatible with the equations of Reference 10, which are the currently baselined Shuttle powered flight guidance equations.* The author feels that second order accuracy is adequate to perform the RTLS powered flight maneuver.

These equations also supercede the equations presented in Section 3.0 of Reference 12. In Reference 12 it was assumed that $T \equiv \underline{\lambda} \cdot \underline{\lambda} = 0$. However, since the writing of Reference 12, the author feels that the steering rate must modulate in direction as well as magnitude (i.e., $T \neq 0$). The value of T is determined to satisfy velocity and position simultaneously.

Dropping third order terms, the steering equations from Section 3.0 (equations 12 and 13) are written as

$$L_p \underline{\lambda} + J_p \dot{\underline{\lambda}} = \underline{V}_{GN} \quad (46)$$

$$S_p \underline{\lambda} + Q_p \dot{\underline{\lambda}} = \underline{R}_{GN} \quad (47)$$

where

$$\underline{V}_{GN} = \underline{V}_{GN} + \underline{V}_{BIAS} \quad (\underline{V}_{BIAS} \text{ is defined on page 31})$$

$$L_p = L - T J_1 - .5 \dot{\underline{\lambda}}^2 J_2 \quad (48)$$

$$S_p = S - T Q_1 - .5 \dot{\underline{\lambda}}^2 Q_2 \quad (49)$$

$$J_p = J_1 - T J_2 \quad (50)$$

$$Q_p = Q_1 - T Q_2 \quad (51)$$

where $\dot{\underline{\lambda}}^2 = \underline{\dot{\lambda}} \cdot \underline{\dot{\lambda}}$.

The above equations are simplified by the original LTG constraint

$$J_1 = J - LK = 0, \text{ or } K = J/L.$$

The above relationship is defined on page A-4 of Appendix A. Equations 48 and 50 become

* The forerunner (presented in Reference 14) of the current steering law was developed as a second order algorithm.

$$L_p = L - .5\dot{\lambda}^2 J_2 \quad (48)$$

$$J_p = -TJ_2 \quad (50)$$

With the proper substitution equations, 46 and 47 become

$$(L - .5\dot{\lambda}^2 J_2) \underline{\lambda} + (-TJ_2) \dot{\underline{\lambda}} = \underline{V}_{GN} \quad (46)$$

$$(S - TQ_1 - .5\dot{\lambda}^2 Q_2) \underline{\lambda} + (Q_1 - TJ_2) \dot{\underline{\lambda}} = f \underline{R}_{GN}, \quad (47)$$

where f is a scalar quantity to enforce downrange position.

Performing the vector scalar product of $\underline{\lambda}$ with equations 46 and 47 it follows that

$$\dot{\lambda}^2 + 2T^2 = 2 (L - \underline{\lambda} \cdot \underline{V}_{GN}) / J_2 \equiv \omega_p^2 \quad (52)$$

$$S - .5(\dot{\lambda}^2 + 2T^2)Q_2 \equiv S - .5\omega_p^2 Q_2 = f \underline{\lambda} \cdot \underline{R}_{GN} \quad (53)$$

The value of ω_p^2 is a well defined quantity and is the first RTLS steering computation, i.e.,

$$\omega_p^2 = 2(L - \underline{\lambda} \cdot \underline{V}_{GN}) / J_2 \quad (54)$$

The present estimates of \underline{R}_{GN} and $\underline{\lambda}$ are available and \underline{R}_{GN} is updated as

$$f = (S - .5\omega_p^2 Q_2) / (\underline{\lambda} \cdot \underline{R}_{GN}) \text{ (from equation 53)}$$

$$\underline{R}_{GN} = f \underline{R}_{GN} \quad (55)$$

Equation 55 provides a stable and accurate prediction of \underline{R}_{GN} , for a fixed thrust level.

Two relationships come from equation 52: assuming that the value of T is known,

$$\dot{\lambda}^2 = \omega_p^2 - 2T^2, \text{ and} \quad (56)$$

assuming that the value of $\dot{\lambda}^2$ is known,

$$T = \pm [.5(\omega_p^2 - \dot{\lambda}^2)]^{1/2} \quad (57)$$

If both T and $\dot{\lambda}^2$ are known, the values of L_p , S_p , J_p , and Q_p are determined from equations 48 through 51.

Now, equations 46 and 47 (incorporating equation 55) are solved simultaneously for $\underline{\lambda}$ and $\dot{\underline{\lambda}}$, i.e.,

$$\underline{\lambda} = \text{Unit} (\underline{V}_{GN} - \underline{R}_{GN} J_p / Q_p), \text{ and} \quad (58)$$

$$\dot{\underline{\lambda}} = (\underline{R}_{GN} - S_p \underline{\lambda}) / Q_p \quad (59)$$

So, the value of $\dot{\lambda}^2$ from the position equation (equation 47) is

$$\dot{\lambda}^2 = \dot{\underline{\lambda}} \cdot \dot{\underline{\lambda}} \quad (60)$$

and the desired unit thrust vector is

$$\underline{i}_f = \text{Unit} [(1 - T x - .5 \dot{\lambda}^2 x^2) \underline{\lambda} + (x - T x^2) \dot{\underline{\lambda}}], \quad (61)$$

where $x = \Delta t - K$, and Δt is the time increment since the present values of $\underline{\lambda}$ and $\dot{\underline{\lambda}}$ were computed.

It is obvious that to have a solution the values of $\dot{\lambda}^2$ from equations 56 and 60 must be equal, i.e.,

$$\dot{\underline{\lambda}} \cdot \dot{\underline{\lambda}} \equiv \dot{\lambda}^2 = \omega_p^2 - 2T^2$$

This relationship can be forced in an iterative manner, e.g.;

$$\dot{\lambda}^2 = .5(\omega_p^2 - 2T^2 + \dot{\lambda}_p^2) \quad (62)$$

where $\dot{\lambda}_p^2 \equiv \dot{\underline{\lambda}} \cdot \dot{\underline{\lambda}}$ (equation 60).

Given this value of $\dot{\lambda}^2$, the value of T can be determined as

$$T = -[.5(\omega_p^2 - \dot{\lambda}^2)]^{1/2} \text{ (equation 57)} \quad (63)$$

It is obvious from equation 57 that two solutions exist to this problem: a positive value of T , and a negative value. The minus sign in equation 63 is arbitrarily chosen.

Values of T and $\dot{\lambda}_p^2$ must be assigned in order to start the algorithm. Arbitrary values are:

$$T = 0 \text{ and } \dot{\lambda}_p^2 = \omega_p^2 \quad (64)$$

Using the above equations, the solution can proceed in the following order:

1. ω_p^2 (equation 54).
2. If the first RTLS guidance pass, $T = 0$, and $\lambda_p^2 = \omega_p^2$
3. $\dot{\lambda}^2$ (equation 62)
4. T (equation 63)
5. S_p , J_p , and Q_p (equations 49, 50, and 51)
6. R_{GN} (equation 55)
7. $\underline{\lambda}$ (equation 58)
8. $\dot{\underline{\lambda}}$ (equation 59)
9. $\dot{\lambda}^2$ (equation 60)
10. \underline{i}_f (equation 61)
11. $\dot{\lambda}_p^2 = \dot{\lambda}^2$ (for next guidance pass)

If the steering vector routine is implemented in the proper manner, steps 1 through 4 are the only additional steps required for powered flight RTLS capability (as in Section 6.0).

6.0 GUIDANCE IMPLEMENTATION

In this section, emphasis is placed on implementation of the Return to Launch Site (RTLS) "single maneuver" abort option. Repeated reference will be made to Reference 10, the currently baselined guidance equations, since the RTLS equations presented here are tailored to be compatible with the Reference 10 equations. The intent is to implement this RTLS option in the SSFS deck in which Rockwell incorporated the Reference 10 equations.

The nomenclature used in Reference 10 (different than that in this report) is defined here:

$t_{go} = T_{GO}$	$r_{grav} = R_g$
$r = R$	$v_{grav} = V_g$
$v = V$	$v_{gn} = V_{GN}$
$r_{go} = R_{GN}$	$P = Q_2$
$v_{go} = V_{GO}$	$H = J_2$
$r_d = R_D$	$Q = Q_1$
$v_d = V_D$	$-TH = J_p$
	$L - .5\lambda^2 H = L_p$

The various maneuver modes are defined in Reference 10:

- $S_{mode} = 1$ Standard ascent
- $S_{mode} = 2$ Reference trajectory - range throttling
- $S_{mode} = 3$ Intercept
- $S_{mode} = 4$ Gravity turn/apsis control (OMS ascent)
- $S_{mode} = 5$ Return to Launch Site (RTLS) - range throttling

The changes to the Reference 10 equations necessary to implement the RTLS option will be listed block by block referring to the flow diagrams of Section 5 (Reference 10).

Block 1 Initialization

(1) After $\underline{r}_{\text{grav}}$ is defined,

$$\underline{v}_{\text{grav}} = 2\underline{r}_{\text{grav}}$$

$$T = 0$$

$$\dot{\lambda} = 0$$

(2) In $S_{\text{mode}} = 5$ initialization (RTLS)

after \underline{r}_d is defined,

$$\underline{i}_x = \text{unit}(\underline{r}_d)$$

Do Block 3.521 \underline{v}_d equation

$$\underline{v}_d = \underline{v}_d + \underline{\omega}_e \times \underline{r}_d$$

$$I_{\text{pass}} = 1$$

$$t_0 = t$$

Block 3.1 Time-To-Go

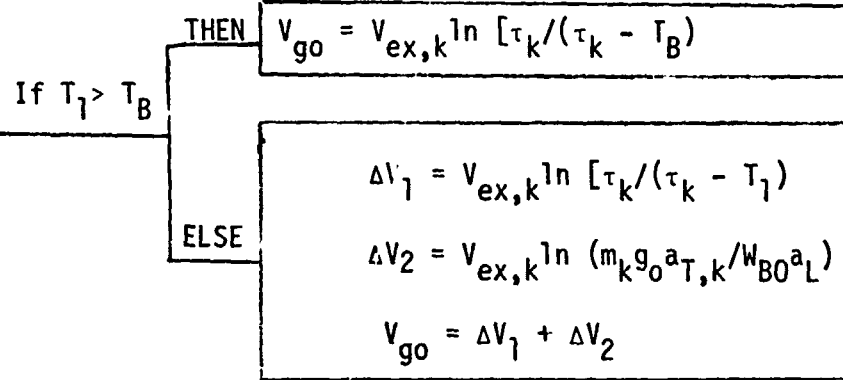
On the line between (Do until $i > n-1$) and (If $n > 1$),

If $S_{\text{mode}} = 5$, do the following computations:

$$T_B = [m_k - (W_{B0}/g_0)]/\dot{m}_k$$

$$T_1 = \tau_k - V_{\text{ex},k}/a_L$$

If $T_1 < 0$, $T_1 = 0$



$$v_{\text{bias}} = B[T_B / (T_B + t - t_0)]^2$$

W_{B0} is input desired burnout weight.

g_0 is reference gravitational acceleration.

B is an input empirical bias (possibly, $B=0$)

t_0 is time RTLS is initiated.

Block 3.2 Integrals

After $K = J/L$

$$P = P - 2QK + SK^2$$

$$H = H - JK$$

$$Q = Q - SK$$

Block 3.3 Turning Rate

If $v_{go} \neq 0$, $\underline{\lambda} = \text{unit } (\underline{v}_{go})$

If $S_{mode} = 1, 2, 3$, $\omega_p^2 = \dot{\lambda}^2$

If $S_{mode} = 4$, $\dot{\lambda} = \mu/vr^2$

$$\underline{r}_{go} = \underline{r}_d - \underline{r} - \underline{v}t_{go} - \underline{r}_{grav} (t_{go}/t'_{go})^2$$

$$\underline{v}_{gn} = \underline{v}_d - \underline{v} - \underline{v}_{grav} (t_{go}/t'_{go})$$

If $I_{pass} = 1$, $\underline{\lambda} = \text{unit } (\underline{v}_{gn})$

$$\underline{v}_{go} = L\underline{\lambda}$$

$$\underline{v}_{gn} = \underline{v}_{gn} + v_{bias} \text{ unit } (\underline{v}_{gn})$$

If $S_{mode} = 5$, $\rightarrow \omega_p^2 = 2(L - \underline{\lambda} \cdot \underline{v}_{gn})/H$

If $I_{pass} = 1$, $\dot{\lambda} = (\omega_p^2)^{1/2}$

$$I_{pass} = 2$$

$$\dot{\lambda} = [.5 (\omega_p^2 - 2T^2 + \dot{\lambda}^2)]^{1/2}$$

If $\dot{\lambda}^2 > \omega_p^2$, $\dot{\lambda} = (\omega_p^2)^{1/2}$

$$T = -[.5 (\omega_p^2 - \dot{\lambda}^2)]^{1/2}$$

Note: ω_p^2 is a quantity,
not the square of
a quantity.

$$S_p = S - TQ - .5\dot{\lambda}^2P$$

$$Q_p = Q - TP$$

If $S_{mode} = 4$, $\underline{r}_{go} = S_p \underline{\lambda} + \dot{\lambda} Q_p \text{ unit } [\underline{\lambda} - (\underline{r}_{go}/\underline{\lambda} \cdot \underline{r}_{go})]$

If $S_{mode} = 1, 2, 3, 5$, $\underline{r}_{go} = [(S - .5 \omega_p^2 P)/(\underline{\lambda} \cdot \underline{r}_{go})] \underline{r}_{go}$

If $Q_p = 0$, $Q_p = 10^6$

If $S_{mode} = 5$, $\underline{\lambda} = \text{unit } (\underline{v}_{gn} + T \underline{r}_{go} H/Q_p)$

$$\dot{\underline{\lambda}} = (\underline{r}_{go} - S_p \underline{\lambda})/Q_p$$

$$\dot{\lambda} = |\dot{\underline{\lambda}}|$$

$$\underline{i}_f = \text{unit } [(1 + TK - .5 \dot{\lambda}^2 K^2) \underline{\lambda} - (K + TK^2) \dot{\underline{\lambda}}]$$

If $S_{mode} \neq 5$, $\omega_p^2 = \dot{\lambda}^2$

Block 3.4 Predictor

$$\underline{v}_{\text{thrust}} = (L - .5 \dot{\lambda}^2 H) \underline{\lambda} - TH \dot{\lambda}$$

$$\underline{r}_{\text{thrust}} = \underline{r}_{\text{go}}$$

Eliminate the $\underline{r}_{\text{bias}}$ computation.

Block 3.53 Range Throttling

$$v_{\text{goz}} = L - .5 \omega_p^2 H$$

If $S_{\text{mode}} = 2$ or 5 (i.e., only one $\Delta r_{\Delta t \text{go}}$ computation),

$$\Delta r_{\Delta t \text{go}} = \underline{\lambda} \cdot \underline{v}_d - .5 v_{\text{goz}}$$

$$\Delta r_z = \underline{\lambda} \cdot (\underline{r}_d - \underline{r} - \underline{v} t_{\text{go}} - \underline{r}_{\text{grav}}) - (S - .5 \omega_p^2 P)$$

7.0 REFERENCES

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APPENDIX A

GENERALIZED THRUST INTEGRALS

Derivation and implementation of the LTG guidance concept requires the definition and evaluation of the following integrals:

$$J_{p_{mi}} = \int_{T_{0i}}^{T_{Fi}} a_i (S-K)^m dS \quad \equiv \quad \int_0^{T_{Bi}} a_i (t+T_{0i}-K)^m dt$$

$$Q_{p_{mi}} = \int_{T_{0i}}^{T_{Fi}} \left[\int_{T_{0i}}^S a_i (S-K)^m dS \right] dt \quad \equiv \quad \int_0^{T_{Bi}} \left[\int_0^t a_i (S+T_{0i}-K)^m dS \right] dt$$

$$t = S - T_{0i}$$

$$m = 0, 1, \dots, 3$$

$$i = K_S, \dots, N_S$$

Where m denotes the order of the integral, K_S is the number of the first thrust phase and N_S is the number of the last thrust phase.

$a_i = (F/M)_i$ = Thrust acceleration of phase i

T_{0i} = Initial time of phase i

T_{Fi} = Final time of phase i

T_{Bi} = Burn time of phase i ($T_{Bi} = T_{Fi} - T_{0i}$)

t and S are variables of integration

K is the time about which the LTG expansion is made.

(K is approximately the midpoint of the maneuver.)

Evaluation of K will be shown later in this appendix.

For the Space Shuttle vehicle it is assumed that either a_i (thrust acceleration) is constant or F_i (thrust) is constant, therefore, these integrals can be evaluated in closed form. After the above integrals are evaluated the total integrals of phase i are evaluated as:

$$J_{mi} = J_{p_{mi}} \quad m = 0, 1, \dots, 3$$

$$Q_{mi} = Q_{p_{mi}} + J_{o_{mi}} T_{Bi}$$

$$\text{Where } J_{o_{mi}} = \sum_{j=K_S}^{i-1} J_{mj}$$

Then the total integrals for the maneuver are:

$$J_m = \sum_{i=K_S}^{N_S} J_{mi} \quad m = 0, 1, \dots, 3$$

$$Q_m = \sum_{i=K_S}^{N_S} Q_{mi}$$

This results in a total of eight integrals.

For constant thrust, the acceleration of phase i is of the form:*

$$a_i = \frac{F}{m_0 - \dot{m}(S - T_{0_i})} = \frac{V_{e_i}}{\tau_i - t}$$

Where $t = S - T_{0_i}$

F = Constant thrust

m_0 = Initial mass

\dot{m} = Constant mass flow rate

V_{e_i} = Engine exhaust velocity

$$\tau_i = m_0 / \dot{m}$$

*For convenience, the subscript "i" is dropped on F , m_0 and \dot{m} .

The first constant thrust integral is:

$$J_{p_{mi}} = \int_0^{T_{B_i}} \frac{Ve_i(t+T_{o_i}-K)^m}{\tau_i-t} dt \quad i = 0, 1, \dots, 3$$

The following identity makes all the integrals recursive in terms of $J_{p_{oi}}$ and $Q_{p_{oi}}$

$$\int_0^{T_{B_i}} \frac{Ve_i(t+T_{o_i}-K)^m}{\tau_i-t} dt = (\tau_i+T_{o_i}-K) \int_0^{T_{B_i}} \left(\frac{Ve_i(t+T_{o_i}-K)^{m-1}}{\tau_i-t} \right) dt - \left[\frac{Ve_i(t+T_{o_i}-K)^m}{m} \right]_0^{T_{B_i}}$$

Therefore,

$$J_{p_{mi}} = (\tau_i+T_{o_i}-K)J_{p_{(m-1)i}} - Ve_i \left[\frac{(T_{B_i}+T_{o_i}-K)^m - (T_{o_i}-K)^m}{m} \right] \quad (1)$$

$$Q_{p_{mi}} = (\tau_i+T_{o_i}-K)Q_{p_{(m-1)i}} - \frac{Ve_i}{m} \left[\frac{(T_{B_i}+T_{o_i}-K)^{m+1} - (T_{o_i}-K)^{m+1}}{m+1} - (T_{o_i}-K)^m T_{B_i} \right]. \quad (2)$$

The integrals are recursive in terms of:

$$L_{p_i} \equiv J_{p_{oi}} = \int_0^{T_{B_i}} \frac{Ve_i dt}{\tau_i-t} = Ve_i \ln \left(\frac{\tau_i}{\tau_i-T_{B_i}} \right) \text{ and}$$

$$S_{p_i} \equiv Q_{p_{oi}} = \int_0^{T_{B_i}} \left[\int_0^t \frac{Ve_i dS}{\tau_i-S} \right] dt = (T_{B_i}-\tau_i)L_{p_i} + Ve_i T_{B_i},$$

since $(t+T_{o_i}-K)^{m=0} = 1$.

For constant acceleration $a_i = a_{L_i} = \text{constant}$ and

$$J_{P_{mi}} = \int_0^{T_{B_i}} a_{L_i} (t + T_{O_i} - K)^m dt$$

$$J_{P_{mi}} = a_{L_i} \left[\frac{(T_{B_i} + T_{O_i} - K)^{m+1} - (T_{O_i} - K)^{m+1}}{m+1} \right] \quad (3)$$

$$Q_{P_{mi}} = \frac{a_{L_i}}{m+1} \left[\frac{(T_{B_i} + T_{O_i} - K)^{m+2} - (T_{O_i} - K)^{m+2}}{m+2} - (T_{O_i} - K)^{m+1} T_{B_i} \right] \quad (4)$$

$m = 0, 1, \dots, 3.$

$$L_{P_i} \equiv J_{P_{oi}} = a_{L_i} T_{B_i}$$

$$S_{P_i} \equiv Q_{P_{oi}} = .5 a_{L_i} T_{B_i}^2 = .5 L_{P_i} T_{B_i}$$

K is evaluated from the LTG constraint

$$J - LK \equiv J_1 = 0.$$

From above equations

$$\begin{aligned} J_1 &= \sum_{i=K_s}^{N_s} J_{P_{1i}} = \sum_{i=K_s}^{N_s} \int_0^{T_{B_i}} a_i (t + T_{O_i} - K) dt \\ &= \sum_{i=K_s}^{N_s} J_{P_i} - K \sum_{i=K_s}^{N_s} L_{P_i} \equiv J - LK, \end{aligned}$$

from which $K = J/L$. (J_{P_i} is defined on next page).

For constant thrust

$$J_{P_i} = \int_0^{T_{B_i}} \frac{V_{e_i}(t+T_{O_i})}{\tau_i - t} dt = (\tau_i + T_{O_i})L_{P_i} - V_{e_i}T_{B_i}$$

and for constant acceleration

$$\begin{aligned} J_{P_i} &= \int_0^{T_{B_i}} a_{L_i}(t+T_{O_i}) dt = .5a_{L_i}T_{B_i}^2 + a_{L_i}T_{O_i}T_{B_i} \\ &= L_{P_i}(.5T_{B_i} + T_{O_i}) \end{aligned}$$

Where $L_{P_i} = a_{L_i}T_{B_i}$.

K is evaluated as above before the remaining integrals are evaluated, since K is needed in the integrals where $m > 0$.

From inspection of equations 1, 2, 3 and 4, it is seen that the constant thrust integrals and the constant acceleration integrals can be put in the same form by making the following definitions.

If Constant Thrust	If Constant Acceleration
$E = 0$	$E = 1$
$F = \tau_i - T_{O_i} - K$	$F = 0$
$A = -V_{e_i}$	$A = a_{L_i}$

$$P'_1 = (T_{B_i} + T_{O_i} - K)^{m+E}$$

$$P'_2 = (T_{O_i} - K)^{m+E}$$

In the following equations, the $m+1$ subscript is used to avoid the zero subscript, for computational purposes (e.g., $J_{P_{0i}} \equiv J_{P_{1i}}$, ..., $J_{P_{3i}} \equiv J_{P_{4i}}$).

$$j_{P(m+1)i} = Fj_{P_{mi}} + A\left(\frac{P'_1 - P'_2}{m+E}\right) \quad (5)$$

$$P_3 = P'_2$$

$$P_1 = P'_1 (T_{B_i} + T_{O_i} - K)$$

$$P_2 = P'_2 (T_{O_i} - K)$$

$$Q_{P(m+1)i} = FQ_{P_{mi}} + \left(\frac{A}{m+E}\right) \left[\frac{P_1 - P_2}{m+E+1} - P_3 T_{B_i} \right] \quad (6)$$

$$m = 1, \dots, 3$$

The following integrals are also used

$$L \equiv J_0 = \sum_{i=K_S}^{N_S} L_{P_i}$$

$$S \equiv Q_0 = \sum_{i=K_S}^{N_S} \left(S_{P_i} + T_{B_i} \sum_{j=K_S}^{i-1} L_{P_j} \right)$$

APPENDIX B

GRAVITY INTEGRALS

A simple, untested gravity model is presented here. The model is probably adequate for all Shuttle maneuvers, and could have advantages over existing models.

Assume that a quantity (e.g., radius vector) can adequately be represented as a third order polynomial in time.

$$\underline{R}(t) = \underline{R} + \underline{V}t + 1/2 \underline{\dot{V}}t^2 + 1/6 \underline{\ddot{V}}t^3 \quad (1)$$

Differentiation yields

$$\underline{V}(t) = \underline{V} + \underline{\dot{V}}t + 1/2 \underline{\ddot{V}}t^2 \quad (2)$$

Initial conditions \underline{R} and \underline{V} are known as well as final conditions \underline{R}_T , \underline{R}_{T_0} and \underline{V}_T . Substituting final condition into equations 1 and 2, coefficients $\underline{\dot{V}}$ and $\underline{\ddot{V}}$ are readily obtained.

Using these coefficients and defining

$$\underline{R}_R = \frac{2}{T_{GO}^2} \int_0^{T_{GO}} \int_0^t \underline{R}(t) dt dt, \quad \underline{R}_V = \frac{1}{T_{GO}} \int_0^{T_{GO}} \underline{R}(t) dt$$

it follows that

$$\underline{R}_V = \underline{R} + 1/2 \underline{V} T_{GO} + 1/2 (\underline{R}_T - \underline{R} - \underline{V} T_{GO}) - 1/12 (\underline{V}_T - \underline{V}) T_{GO}$$

and

$$\underline{R}_R = \underline{R} + 1/3 \underline{V} T_{GO} + 3/10 (\underline{R}_T - \underline{R} - \underline{V} T_{GO}) - 1/15 (\underline{V}_T - \underline{V}) T_{GO}.$$

Rearranging these equations results in

$$\underline{R}_V = 1/2 (\underline{R}_T + \underline{R}) - 1/12 (\underline{V}_T - \underline{V}) T_{GO} \quad (3)$$

$$\underline{R}_R = 1/10 (3\underline{R}_T + 7\underline{R}) - 1/15 (\underline{V}_T - 3/2 \underline{V}) T_{GO} \quad (4)$$

These are mean values of \underline{R} to be used in gravity computations since gravity can be approximated as:

* For convenience, parentheses are omitted on fractions in the following equations in this section, e.g., $3/2 \underline{V} \equiv (3/2) \underline{V}$.

$$\underline{G}(t) = \frac{\mu}{R_c^3} \underline{R}(t), \text{ where } \mu \text{ and } R_c \text{ are constants and}$$

$$\underline{G}_V T_{GO} \equiv \int_0^{T_{GO}} \underline{G}(t) dt \text{ and } 1/2 \underline{G}_R T_{GO}^2 \equiv \int_0^{T_{GO}} \int_c^t \underline{G}(t) dt dt .$$

From the above equations it is seen that

$$\underline{G}_V = \frac{\mu}{R_V^3} \underline{R}_V \text{ and } \underline{G}_R = \frac{\mu}{R_R^3} \underline{R}_R \text{ are good approximations and gravity effects}$$

are expressed as

$$\underline{V}_g = \underline{G}_V T_{GO} \text{ and } \underline{R}_g = 1/2 \underline{G}_R T_{GO}^2 .$$

However, simulations were conducted by the Guidance and Dynamics Branch of MPAD* using the above model. It was determined that deriving \underline{V}_g and \underline{R}_g in cartesian coordinates as above is not adequate for long, low thrust burn arcs such as Shuttle de-orbit.

The above model is quite accurate for short, high thrust burn arcs; however, the "generalized" gravity model presented here makes use of polar coordinates in order to more adequately handle long burn arcs. It could be thought of as a third order "conic + thrust" solution. Using the method described above, the gravity model is as follows.

Transforming initial and final state to polar coordinates

$$\underline{p} = \begin{bmatrix} R \\ \sin^{-1}(\underline{x}_L \cdot \underline{u}_y) \\ \sin^{-1}(\underline{x}_L \cdot \underline{u}_z) \end{bmatrix} \equiv \begin{bmatrix} R \\ \psi \\ \phi \end{bmatrix}, \quad \dot{\underline{p}} = \begin{bmatrix} \underline{V} \cdot \underline{R} \\ \underline{V} \cdot \underline{Y}_L \\ \underline{V} \cdot \underline{Z}_L \end{bmatrix} \frac{1}{R} \equiv \begin{bmatrix} \dot{R} \\ \dot{\psi} \\ \dot{\phi} \end{bmatrix}, \text{ where } \begin{array}{l} \underline{x}_L = \text{unit } (\underline{R}) \\ \underline{z}_L = \text{unit } (\underline{x}_L \times \underline{u}_y) \\ \underline{y}_L = \underline{z}_L \times \underline{x}_L \end{array}$$

$$\underline{p}_T = \begin{bmatrix} R_D \\ 0 \\ 0 \end{bmatrix} \equiv \begin{bmatrix} R_T \\ \psi_T \\ \phi_T \end{bmatrix}, \quad \dot{\underline{p}}_T = \begin{bmatrix} \underline{V}_T \cdot \underline{R}_T \\ 0 \\ \underline{V}_T \cdot \underline{u}_z \end{bmatrix} \frac{1}{R_D} \equiv \begin{bmatrix} \dot{R}_T \\ \dot{\psi}_T \\ \dot{\phi}_T \end{bmatrix}$$

Expressing the mean values in polar coordinates (equations 3 and 4):

$$\underline{p}_V = 1/2 (\underline{p}_T + \underline{p}) - 1/12 (\dot{\underline{p}}_T - \dot{\underline{p}}) T_{GO}, \text{ and}$$

$$\underline{p}_R = 1/10 (3\dot{\underline{p}}_T + 7\dot{\underline{p}}) - 1/15 (\underline{p}_T - 3/2 \underline{p}) T_{GO} .$$

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Transforming polar mean values to cartesian coordinates.

$$\underline{R}_V = [\cos p_{V_2} (\underline{U}_X \cos p_{V_3} + \underline{U}_Z \sin p_{V_3}) + \underline{U}_Y \sin p_{V_2}] p_{V_1}, \text{ and}$$

$$\underline{R}_R = [\cos p_{R_2} (\underline{U}_X \cos p_{R_3} + \underline{U}_Z \sin p_{R_3}) + \underline{U}_Y \sin p_{R_2}] p_{R_1}.$$

Gravity losses are now expressed as follows:

$$\underline{G}_V = f(\underline{R}_V), \quad \underline{V}_g = \underline{G}_V^T \underline{G}_0$$

$$\underline{G}_R = f(\underline{R}_R), \quad \underline{R}_g = 1/2 \underline{G}_R^T \underline{G}_0^2$$

where $f(\underline{R})$ is an accurate gravity model or a simple spherical gravity model (i.e., $\underline{G} = -\frac{1}{R^3} \underline{R}$). It is possible that an accurate gravity model is

more adequate for low thrust (OMS) burns (for accurate state extrapolation).